Robust Estimation in Autoregressive Models with Skewed Innovations

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Background: Traditional estimation methods for autoregressive (AR) time series models, such as Ordinary Least Squares (OLS) and Yule-Walker (YW), fundamentally rely on the assumption of independent, identically distributed (iid) normal innovations. While this assumption is often adopted for theoretical simplicity, it rarely holds true in empirical applications—particularly with environmental datasets. Environmental time series frequently exhibit severe skewness, heavy tails, and structural outliers, all of which result in significantly biased and statistically inefficient estimates. While robust alternatives like M-estimators are common in other statistical contexts, their performance in time series models is often hindered by a low breakdown point. This occurs because heavy-tailed innovations create a "double effect": they act as outliers in the current period and propagate as leverage points in subsequent lagged variables. Unlike many other robust approaches, the Modified Maximum Likelihood (MML) method builds directly on the innovation distribution to provide explicit estimators.

Objectives: This project evaluates the performance of MML estimators against other outlier-resistant modifications, such as GM-estimators and MM-estimators under skewed innovations—a comparison that has not been previously explored. Our study provides a framework for more accurate evaluations of environmental time series data.

Methods: Using a series of Monte Carlo simulations, we will compare the performance of MML estimators with that of conventional and robust estimators under different settings, including multiple autoregressive model orders, various shape parameters, and different sample sizes.

Results: MML estimators showed minimal bias and the lowest mean squared error. Other estimators were biased for at least one model parameter and were less efficient compared to MML.

Conclusions: Our simulations showed that MML is the only method that is unbiased and asymptotically efficient across all simulation settings.

Recommendations: The results emphasize the importance of checking the normality assumption and using appropriate robust methods, as conventional estimators become biased when innovations are skewed.