Heteroscedastic ANOVA - old solutions, new views

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Abstract

The generalization of the Behrens-Fisher problem to comparing $k > 2$ means from nonhomogenous populations has attracted the attention of statisticians for many decades. Several approaches offer different approximations to the distribution of the test statistic. The question of statistical properties of these approximations is still alive. One is the Fai-Cornelius generalization of Satterthwaite's approximation of degrees of freedom. As it turns out, this approximation is dependent on the choice of the hypothesis contrast matrix. Here we present a brief overview of several approaches suggested in the literature with a focus on investigating the Satterthwaite-Fai- Cornelius approximation. We illustrate by simulation the behavior of p-values under the null hypothesis. In addition to the Satterthwaite-Fai-Cornelius test, the Kenward-Roger test, the simple ANOVA F-test, the parametric bootstrap test, and the generalized F-test will be briefly discussed.

Keywords ANOVA model, fixed effects, heterogeneous variance

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