

A study of multilevel models with block circular symmetric covariance structures

Yuli Liang

Department of Statistics, Stockholm University, Stockholm, Sweden
email:yuli.liang@stat.su.se

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Abstract.

Our work concerns the study of multilevel models with specific patterned covariance structures and addresses the issues of maximum likelihood estimation. In particular, circular symmetric hierarchical data structures are considered. Models which covariance structures reflect both circularity and exchangeability present in the data can be widely used in different applications, with early examples from psychometric and medical research.

Two derived patterns of the covariance matrices which characterizes models under consideration. The relationship between these two patterned covariance matrices was investigated and it has been verified they are similar matrices. New expressions for the eigenvalues of block circular symmetric matrices are obtained which take into account the block structure. Maximum likelihood estimation of balanced multilevel models with block circular symmetric covariance matrices is discussed. We show that explicit maximum likelihood estimators of variance components exist under certain restrictions on the parameter space.

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