

Outline

- The Wilcoxon Signed Rank Test
- The Wilcoxon Rank Sum Test

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The Wilcoxon Signed Rank Test

- For continuous data
- Regression to the mean Example (pp. 253-255)
 - Data on 14 boys with extreme “day 1” values
 - Change in direction of mean (+)
 - Change away from mean (-)
 - If same value, no sign assigned (excluded from analysis)
 - Rank absolute differences (all, ascending)
 - Sum ranks separately (+/-)
 - Use R_{WSR} : signed rank sum for positive differences (those changing toward the mean) as test statistic

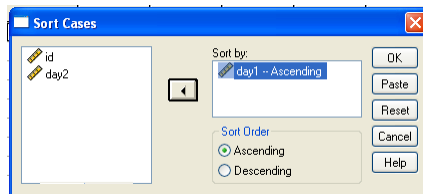
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How to in SPSS

1. Open file (Excel, first row variable names)
2. Sort by day1 (Data → Sort Cases...)
3. Clear rows 8-26 (Hold-down left click on row 8, drag down to row 26 →Edit→Clear)
4. Rank by day1 (Transform → Rank Cases)
5. Sort by id (Data→Sort Cases)
6. Create "toward" (Transform→Compute→'Target Variable' toward→"If" ::: →'Numeric Expression')
7. Create "away" (ibid but :::)
8. Create rtm (same as "toward" and "away" but on same column)
9. Find ranks by rtm (Transform → Rank Cases)
10. Create +/- groups (Transform→Compute)
11. Sums from group ranks (Analyze→Reports→Report Summaries in Columns)

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2. Sort cases by day1

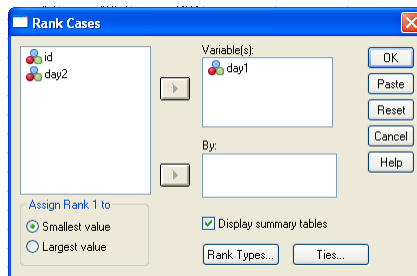


3. Clear rows 8-26

The screenshot shows the SPSS Data Editor with a context menu open over rows 8-26. The 'Clear' option is selected. The data table is visible in the background.

	day2	var	var	var
8	1054			
9	1844			
10	1623			
11	2907			
12	1236			
13	1797			
14	1748			
15	1566			
16	1905			
17	2074	3312		
18	2301	4120		
19	2310	1569		
20	2330	2339		
21	2340	3182		
22	2348	2122		
23	2400	2554		
24	2436	2189		
25	2546	1732		
26	2594	2867		
27	2685	2304		
28	2748	2104		
29	130	2773	3236	

4. Rank data by day1



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6. Toward

The image shows two overlapping SPSS dialog boxes. The background box is the 'Compute Variable' dialog, and the foreground box is the 'Compute Variable: If Cases' dialog.

Compute Variable Dialog:

- Target Variable: toward
- Numeric Expression: $\text{abs}(\text{day1} - \text{day2})$
- Functions and Special Variables: (Empty)
- If...: $(\text{Rday1} \leq 7 \ \& \ (\text{day1} < \text{day2})) \ | \ (\text{Rday1} >= 8 \ \& \ (\text{day1} > \text{day2}))$

Compute Variable: If Cases Dialog:

- Include if case satisfies condition: (Selected)
- Condition: $(\text{Rday1} \leq 7 \ \& \ (\text{day1} < \text{day2})) \ | \ (\text{Rday1} >= 8 \ \& \ (\text{day1} > \text{day2}))$
- Function group: All
- Functions and Special Variables: (Empty)

Buttons: OK, Paste, Reset, Cancel, Help (in both dialogs).

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7. Away

The image shows two overlapping SPSS dialog boxes. The background box is the 'Compute Variable' dialog, and the foreground box is the 'Compute Variable: If Cases' dialog.

Compute Variable Dialog:

- Target Variable: away
- Numeric Expression: $\text{abs}(\text{day1} - \text{day2})$
- Functions and Special Variables: (Empty)
- If...: $(\text{Rday1} \leq 7 \ \& \ (\text{day1} > \text{day2})) \ | \ (\text{Rday1} >= 8 \ \& \ (\text{day1} < \text{day2}))$

Compute Variable: If Cases Dialog:

- Include if case satisfies condition: (Selected)
- Condition: $(\text{Rday1} \leq 7 \ \& \ (\text{day1} > \text{day2})) \ | \ (\text{Rday1} >= 8 \ \& \ (\text{day1} < \text{day2}))$
- Function group: All
- Functions and Special Variables: (Empty)

Buttons: OK, Paste, Reset, Cancel, Help (in both dialogs).

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Data after step 10

	day2	Rday1	toward	away	rtm	Rrtm	rtm2	var
1	2484	1.000	1431.00	.	1431.00	13.000	+	
2	2926	14.000	1396.00	.	1396.00	12.000	+	
3	1054	6.000	.	699.00	699.00	10.000	-	
4	3289	13.000	243.00	.	243.00	3.000	+	
5	2849	9.000	.	7.00	7.00	1.000	-	
6	1925	3.000	420.00	.	420.00	4.000	+	
7	2431	11.000	645.00	.	645.00	9.000	+	
8	810	2.000	.	482.00	482.00	7.000	-	
9	2573	10.000	476.00	.	476.00	6.000	+	
10	2185	12.000	1092.00	.	1092.00	11.000	+	
11	1844	7.000	63.00	.	63.00	2.000	+	
12	3236	8.000	.	463.00	463.00	5.000	-	
13	2269	4.000	624.00	.	624.00	8.000	+	
14	3163	5.000	1440.00	.	1440.00	14.000	+	
15								
16								
17								
18								
19								
20								
21								

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11. Sums of group ranks

Report: Summaries in Columns

Data Columns: Rrtm:sum

Break Columns: rtm2 -- Ascending

Sort Sequence: Ascending

Report: Options... Layout... Titles...

Rank of rtm		Sum
rtm2		
-		23.000
+		82.000

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Rationale of test

- For n observations, ranks are $1, \dots, n$
- Sum of ranks is $n(n+1)/2$, and average rank is $(n+1)/2$
- Null hypothesis
- If the null is true, distribution of differences is symmetric
 - Half positive, half negative
- If null hypothesis is true, the sum of the ranks should be $n/2$ times the average rank, i.e. $(n/2)(n+1)/2 = n(n+1)/4$
- Tables 9.3 and 9.4 exemplify the construction of boundaries of the rejection region as shown in Table B9 for $n < 30$ (469):
 - N on first column, then
 - Two sided
 - One sided

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Example 9.3 (p. 255)

- $N=14$, $\alpha = 0.05$ $\alpha = 0.05$
- Null hypothesis $H_0 : M_d = 0$ (median of differences equals zero, no regression toward the mean)
- Alternative $H_0 : M_d > 0$ (median differences larger than zero, regression toward the mean)
- Critical values $(25, 80)$ (if R_{WSR} is larger than or equal to 80, reject)
- Since $R_{WSR} = 82$ we can reject the null
- This result differs with that from Sign test (didn't reject):
 - Wilcoxon Sum Rank test has larger power
 - WSR uses more information, not only counts of changes toward the mean
 - Average rank away=5.75 vs. average rank toward=7.5
 - P-value from sig test is 0.0898, not so far from 0.05

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TIES

When applying WSR, two types of ties can occur:

- The difference is zero (e.g. regression to the mean data) or the observed value equals the hypothesized value (e.g. Example 9.4 next).
 - REMOVE observation(s) from analysis;
 - FEW ties of this type are allowed
- Two or more differences have the same nonzero value.
 - CONVENTION: use average rank (e.g. 2,3→2.5; 2,3,4→3).
 - Few ties of this type: still can use WSR statistic (but results are approximate)
 - Many ties of this type: adjustment for ties needed or next Chapter (contingency table)

$$R_{WSR} \quad H_0 : M_d = 0 \quad R_{WSR} = 82$$
$$(25,80) \quad H_0 : M_d > 0$$

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Example 9.4

DATA: Lead concentration in one lab (n=13)

- Are measurements significantly different from 41?

$$H_0 : M_d = 41 \quad \text{vs.} \quad H_0 : M_d \neq 41 \quad \alpha = 0.05$$

- Eleventh datum dropped, new n=12
- Test statistic $R_{WSR} = 58.5$
- From Table B9 (p. 469): n=12, $\alpha \leq 0.05$, two-sided the boundaries are (13,65)
- $58.5 < 65 \rightarrow$ cannot reject
- WSR decision agrees with sign test

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Difference from 41

Absolute difference from 41

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Define group with positive Differences ...

... for cases with concentration larger than 41

Repeat for concentrations smaller than 41, assign '-'

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Data

Getting sum of ranks by group

ID	Concentration	Difference	AbsDiff	RAbsDiff	Group
1	45.0	4.00	4.00	5.0	+
2	43.0	2.00	2.00	2.0	+
3	40.0	-1.00	1.00	1.0	-
4	44.0	3.00	3.00	3.5	+
5	49.0	8.00	8.00	10.0	+
6	36.0	-5.00	5.00	6.5	-
7	51.0	10.00	10.00	12.0	+
8	46.0	5.00	5.00	6.5	+
9	35.0	-6.00	6.00	8.5	-
10	50.0	9.00	9.00	11.0	+
11	41.0
12	38.0	-3.00	3.00	3.5	-
13	47.0	6.00	6.00	8.5	+

Group	Rank of AbsDiff Sum
-	19.5
+	58.5

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Normal Approximation to WSR

- For $n \geq 16$ R_{WSR} follows approx. a normal distribution
- Can use

$$\frac{|R_{WSR} - \overbrace{[n(n+1)/4]}^{\text{Expected value}}| - 0.5}{\underbrace{\sqrt{n(n+1)(2n+1)/24}}_{\text{Variance}}}$$
 Continuity correction
- For Example 9.3 (recall one-sided)

$$\frac{|82 - [14(14+1)/4]| - 0.5}{\sqrt{14(14+1)(2(14)+1)/24}} = \frac{|82 - 52.5| - 0.5}{15.93} = 1.82$$
- From Table B4, p-value = $P[Z > 1.82] = 0.0344$
- This p-value is close to the exact (0.0338). Approximation is good for this dataset
- Study last paragraph of Section 9.3 (p. 257)**

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The Wilcoxon Rank Sum Test

- *Sign and WSR test for one group or paired data*
- Now assume we have two groups
- WRS is also known as Mann-Whitney (unequal sample sizes)
- Data: proportion of calories from fat in two groups

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1. Type in data (ID, Prop_fat, Grades), for one case
2. Change variable Name
3. Change data Type
4. Change Alignment
5. Rank all data
6. Get sum of ranks per group (Analyze→Reports→Report Summaries in Columns)

ID	Prop_fat	Grades	RProp_fa
1	.365	5-6	21.0
2	.437	5-6	30.0
3	.248	5-6	4.0
4	.424	5-6	26.0
5	.403	5-6	23.0
6	.337	5-6	16.5
7	.295	5-6	11.0
8	.319	5-6	14.0
9	.285	5-6	9.0
10	.465	5-6	32.0
11	.255	5-6	5.0
12	.125	5-6	1.0
13	.427	5-6	29.0
14	.225	5-6	3.0
15	.311	7-8	13.0
16	.278	7-8	6.0
17	.282	7-8	8.0
18	.421	7-8	25.0

Name	Type	Width	Decimals	Label	Values	Missing	Columns	Align	Measure
1 ID	String	2	0		None	None	8	Right	Nominal
2 Prop_fat	Numeric	8	3		None	None	8	Center	Scale
3 Grades	String	3	0		None	None	8	Center	Nominal
4 RProp_fa	Numeric	9	1	Rank of Prop_f	None	None	11	Center	Scale

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- Data for two cases were rounded to allow a tie
- Test statistic is the sum of ranks of smaller sample

$$R_{WRS} = 224.5, \quad n_1 = 14, \quad n = 33$$

Grades	Rank of Prop fat Sum
5-6	224.5
7-8	336.5

- RATIONALE:

- If there were no differences between groups, then the rank sum for the smaller group is the product $n_1(n+1)/2 = 14(33+1)/2 = 238$
- A value of R_{WRS} far away from the expected under the null are grounds for rejecting the hypothesis of no difference
- Use critical values on Table B10 (p. 470-473)
 - N_1 smaller group, N_2 larger group
 - two-sided or one-sided
 - Critical regions are determined similarly to WSR test
 - An example for $n=8, n_1=4$ is described on page 258 and Tables 9.6 and 9.7 show relevant counts and probabilities

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- Example 9.5 (p. 260)

- Null: no difference in proportion of calories from fat in these two groups
- Alternative: there is a difference
 - Two-sided, let's do $\alpha \leq 0.01$
 - Will reject if extremely large or small values of R_{WRS}
- Observed $R_{WRS} = 224.5, \quad n_1 = 14, \quad n_2 = 19$
- From Table B10 (p. 473) critical values are (168,308)
- Reject the null if observed R_{WRS} is smaller than 168 or larger than 308
- $R_{WRS} = 224.5$ is not in the rejection region. Cannot reject the null.
- Conclusion: based on the WRS test there is no evidence that grade groups differ in the proportion of calories from fat

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Normal Approximation to WRS

- If Table B10 cannot accommodate data or $n_1 > 8$ and $n_2 > 8$

- Can use
$$\frac{|R_{WRS} - \overbrace{[n_1(n+1)/2]}^{\text{Expected value}}| - 0.5}{\underbrace{\sqrt{n_1 n_2 (n+1)/12}}_{\text{Variance}}}$$
 Continuity correction

- For Example 9.3 (recall two-sided) $R_{WRS} = 224.5$, $n_1 = 14$, $n_2 = 19$

$$\frac{|224.5 - [14(33+1)/2]| - 0.5}{\sqrt{14(19)(33+1)/12}} = \frac{|224.5 - 238| - 0.5}{27.453} = 0.4735$$

- From Table B4, p-value = $2P[Z > 0.4735] = 2(0.3174) = 0.63$ (interpolation)
- For many ties use adjustment or next Chapter approach